Nonlinear Analysis of a Colpitts Injection-Locked Oscillator

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Abstract

An analysis of an injection-locked oscillator based on Colpitts circuit which can be used as a frequency divider is proposed. Analytical models for fundamental injection locking (synchronous oscillators) and intermodulation (superharmonic) injection locking (frequency dividers) are derived by employing Bessel functions to account for the exponential nonlinearity existing inside the circuit.

The proposed analytical models match the simulation results obtained from a BJT implementation. The proposed method can be extended to CMOS implementation utilizing MOS transistors operating in weak-inversion region.

1. Colpitts Injection-Locked Oscillator Model

The circuit diagram and the block diagram associated with the phase shift between signals are shown below. The injection signal is applied to the base and the output signal is taken from the collector of the transistor.

From the vector diagram, the base-emitter signal consisting only of the fundamental frequency can be derived from the current-voltage equation of a BJT by using Bessel functions.

\[
v_b(t) = V_c \cdot \cos(\omega_t t + \alpha) + iv_t(t) = 2 \cdot I_c \cdot \frac{I_v(V_c)}{I_v(V_c)} \cos(\alpha \omega t + \alpha)
\]

where:

\[
v_c = V_c = \frac{V_T^2 + V_b^2 - 2 V_b V_c \cos(\phi)}{U_T}
\]

and:

\[
\alpha = -\tan^{-1}(\frac{V_b}{V_c \cos(\phi)})
\]

Then, the output equation can be written:

\[
v_{out} = I_{oI} \cdot Z_r(\alpha)
\]

After substitution and dividing the Imaginary part by the Real part, the maximum relative locking range (the offset relative to the half BW) and the corresponding phase can be derived:

\[
\Delta f_{rel-max (pos. offset)} = \frac{1}{2} \frac{V_o(V_c)}{I_v(V_c)} \Delta f_{rel-max (pos. offset)} = 0.5 \cdot \frac{2}{V_o(V_c)}
\]

where the feedback voltage amplitude at the maximum locking range can be found numerically as follows (lower case letters of the voltage signals represent amplitudes normalized to kT/q).

\[
v^2 = 2 \left( \frac{V_t}{U_T} \right) \int \left( \frac{V_b^2 + V_c^2 - 2 V_b V_c \cos(\phi)}{U_T} \right) \sqrt{V_b^2 - V_c^2} \cdot Z_{2D}
\]

3. Intermodulation Injection Locking

In the intermodulation locking (\(v_b = V_{los} \alpha \omega),\) the frequencies at the base and at the emitter are different, so the signal at each node will be treated by Bessel functions separately in the collector current equation:

\[
i_{e}(t) = I_c \cdot e^{V_{los} \sin(\omega t) + \phi} - e^{V_{los} \sin(\omega t - \phi)}
\]

After approximating each exponential term, the maximum relative locking range equation can be found using the same procedure as in the fundamental locking. It can be simplified for small input and feedback amplitudes as:

\[
\phi_{rel-max} = \frac{\pi}{2} \cdot \frac{V_o(V_c)}{I_v(V_c)}
\]

For divide-by-2 operation, N is set to 2 and the general equations can be simplified to:

\[
\Delta f_{rel-max (pos. offset)} = \frac{1}{2} \frac{V_o(V_c)}{I_v(V_c)} \Delta f_{rel-max (pos. offset)} = 0.5 \cdot \frac{2}{V_o(V_c)}
\]

where the normalized feedback voltage amplitude, \(v_b\) at the maximum locking range can be found from the Real part of the output equation and using the same derivation as in fundamental locking.

4. Results Comparison

The proposed fundamental and intermodulation (divide-by-2) injection locking models are verified by comparing to the SPICE simulated results from a 1-MHz Colpitts injection-locked oscillator via the HFA3046 RF BJT model.

Conclusion

Analytical models for a Colpitts injection-locked oscillator which can be used as a frequency divider are proposed. For the fundamental locking, the locking range depends strongly on both the input and the feedback amplitude hence some discrepancy in the results due to the inaccuracy of the output amplitude calculation. For the divide-by-2 operation, the locking range is mainly dependent of the input amplitude, hence the prediction is quite accurate.