Non-Cooperative Multi-Radio Channel Allocation in Wireless Networks

Márk Félegyházi*, Mario Čagalj†, Shirin Saeedi Bidokhti*, Jean-Pierre Hubaux*

* Ecole Polytechnique Federale de Lausanne (EPFL), Lausanne, Switzerland
† University of Split, Croatia

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Problem

- multi-radio devices
- set of available channels

How to assign radios to available channels?
System model (1/3)

- $X$ – set of orthogonal channels ($|X| = C$)
- $N$ – set of communicating pairs of devices ($|N| = N$)
- sender controls the communication (sender and receiver are synchronized)
- single collision domain if they use the same channel
- devices have multiple radios
- $k$ radios at each device, $k \leq C$
System model (2/3)

- $N$ communicating pairs of devices
- $C$ orthogonal channels
- $k$ radios at each device

$k_{i,x} \rightarrow$ number of radios by sender $i$
on channel $x$

$\sum_{x \in C} k_{i,x}$

$k_x = \sum_{i \in N} k_{i,x}$

example:

Intuition:

Use multiple radios on one channel?
System model (3/3)

- channels with the same properties
- $\tau'(k_x)$ – total throughput on any channel $x$
- $\tau(k_x)$ – throughput per radio
Multi-radio channel allocation (MRCA) game

- selfish users (communicating pairs)
- non-cooperative game $G_{\text{MRCA}}$
  - players $\rightarrow$ senders
  - strategy $\rightarrow$ channel allocation
  - payoff $\rightarrow$ total throughput

- strategy: $s_i = \{k_{i,1}, ..., k_{i,C}\}$

- strategy matrix:
  $$S = \begin{pmatrix}
  s_1 \\
  \vdots \\
  s_N
  \end{pmatrix}$$

- payoff:
  $$u_i = \tau_i = \sum_{x \in C} (k_{i,x} \cdot \tau(k_x))$$

$$\begin{array}{cccccc}
\text{p1} & 1 & 1 & 1 & 1 & 0 & 0 \\
\text{p2} & 1 & 0 & 1 & 1 & 1 & 0 \\
\text{p3} & 1 & 2 & 0 & 0 & 0 & 1 \\
\text{p4} & 1 & 0 & 0 & 1 & 0 & 0 \\
\end{array}$$

$\text{N=4, C=6, k=4}$
Game-Theoretic Concepts

**Best response:** Best strategy of player $i$ given the strategies of others.

$$br_i(s_{-i}) = \left\{ s_i \in S : u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}), \forall s'_i \in S \right\}$$

**Nash equilibrium:** No player has an incentive to unilaterally deviate.

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \forall s_i \in S$$

**Pareto-optimality:** The strategy profile $s^{po}$ is Pareto-optimal if:

$$\exists s^i : u_i(s^i) \geq u_i(s^{po}), \forall i \text{ with strict inequality for at least one player } i$$

**Price of anarchy:** The ratio between the total payoff of players playing a socially-optimal (max. Pareto-optimal) strategy and a worst Nash equilibrium.

$$POA = \frac{\sum_i u_i^{so}}{\sum_i u_i^{w-NE}}$$
Use of all radios

**Lemma:** If $S^*$ is a NE in $G_{MRCA}$, then $k_i = k$, $\forall i$.

Each player should use all of his radios.

**Intuition:** Player $i$ is always better off deploying unused radios.
Load-balancing channel allocation

- Consider two arbitrary channels $x$ and $y$ in $X$, where $k_x \geq k_y$
- Distance: $d_{x,y} = k_x - k_y$

**Proposition:** If $S^*$ is a NE in $G_{MRCA}$, then $d_{y,x} \leq 1$, for any channel $x$ and $y$. 

![Diagram of load-balancing channel allocation]
Nash equilibria (1/2)

- Consider two arbitrary channels $x$ and $y$ in $X$, where $k_x \geq k_y$
- distance: $d_{x,y} = k_x - k_y$

**Theorem 1:** A channel allocation $S^*$ is a Nash equilibrium in $G_{\text{MRCA}}$ if for all $i$:
- $d_{x,y} \leq 1$ and
- $k_{i,x} \leq 1$. 

**Nash Equilibrium:**

Use one radio per channel.

<table>
<thead>
<tr>
<th></th>
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<th>channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>$p_1$</td>
<td>$p_2$</td>
<td>$p_1$</td>
<td>$p_2$</td>
<td>$c_1$</td>
</tr>
<tr>
<td>$p_2$</td>
<td>$p_3$</td>
<td>$p_3$</td>
<td>$p_4$</td>
<td>$p_2$</td>
<td>$c_2$</td>
</tr>
</tbody>
</table>

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Nash equilibria (2/2)

- Consider two arbitrary channels $x$ and $y$ in $X$, where $k_x \geq k_y$
- distance: $d_{x,y} = k_x - k_y$
- loaded and less loaded channels: $X^+$ and $X^-$

**Theorem 2:** A channel allocation $S^*$ is a Nash equilibrium in $G_{MRCA}$ if:

- $d_{x,y} \leq 1$,
- for any player $i$ who has $k_{i,x} \geq 2$, $x$ in $X$, $k_{i,x} \leq \frac{\tau(k_x - 1) - \tau(k_x + 1)}{\tau(k_x - 1) - \tau(k_x)}$
- for any player $i$ who has $k_{i,x} \geq 2$ and $x$ in $X^+$, $k_{i,y} \geq k_{i,x} - 1$, for all $y$ in $X^-$

Use multiple radios on certain channels.
**Efficiency (1/2)**

**Theorem:** In $G_{MRCA}$, the price of anarchy is:

$$POA = \frac{\tau^t(1)}{\left(k_x + 1 - \frac{N \cdot k}{C}\right) \cdot (\tau^t(k_x) - \tau^t(k_x + 1)) + \tau^t(k_x + 1)}$$

where $k_x = \left\lfloor \frac{N \cdot k}{C} \right\rfloor$, $k_x + 1 = \left\lceil \frac{N \cdot k}{C} \right\rceil$

**Corollary:** If $\tau^t(k_x)$ is constant (i.e., ideal TDMA), then any Nash equilibrium channel allocation is Pareto-optimal in $G_{MRCA}$. 
Efficiency (2/2)

- In theory, if the total throughput function $\tau(k_x)$ is constant $\Rightarrow$ POA = 1
- In practice, there are collisions, but $\tau(k_x)$ decreases slowly with $k_x$ (due to the RTS/CTS method)

Summary

- wireless networks with multi-radio devices
- users of the devices are selfish players
- $G_{\text{MRCA}}$ – multi-radio channel allocation game
- results for a Nash equilibrium:
  - players should use all their radios
  - load-balancing channel allocation
  - two types of Nash equilibria
  - NE are efficient both in theory and practice
- fairness issues
- coalition-proof equilibria
- algorithms to achieve efficient NE:
  - centralized algorithm with perfect information
  - distributed algorithm with imperfect information

http://people.epfl.ch/mark.felegyhazi
Future work

- general scenario – conjecture: hard
- approximation algorithms
- extend model to mesh networks (multihop communication)
Extensions
Related work

► Channel allocation
  – in WLANs [Mishra et al. 2005]
  – in cognitive radio networks [Zheng and Cao 2005]

► Multi-radio networks
  – cognitive radio [So et al. 2005]

► Competitive medium access
  – Aloha [MacKenzie and Wicker 2003, Yuen and Marbach 2005]
  – WLAN channel coloring [Halldórsson et al. 2004]
  – channel allocation in cognitive radio networks [Cao and Zheng 2005, Nie and Comaniciu 2005]
Fairness

Nash equilibria (fair)

Nash equilibria (unfair)

Theorem: A NE channel allocation $S^*$ is max-min fair iff

$$\sum_{x \in C_{\text{min}}} k_{i,x} = \sum_{x \in C_{\text{min}}} k_{j,x}, \forall i, j \in N$$

Intuition: This implies equality: $u_i = u_j, \forall i,j \in N$
Centralized algorithm

Assign links to the channels *sequentially*.
Convergence to NE (1/3)

Algorithm with imperfect info:

- move links from “crowded” channels to other randomly chosen channels
- desynchronize the changes
- convergence is not ensured

\[ N = 5, \ C = 6, \ k = 3 \]
Algorithm with imperfect info:

- move links from “crowded” channels to other randomly chosen channels
- desynchronize the changes
- *convergence is not ensured*

Balance: $\beta(S) = \sum_{x \in C} \left| k_x - \frac{N \cdot k}{C} \right|$

Unbalanced (UB): $\beta(UB) = 15$

Best balance (NE): $\beta(UB) = 3$

Efficiency: $\phi(S) = \frac{\beta(S_{UB}) - \beta(S)}{\beta(S_{UB}) - \beta(S_{NE})}$

$0 \leq \phi(S) \leq 1$
Convergence to NE (3/3)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N (# of pairs)</td>
<td>10</td>
</tr>
<tr>
<td>C (# of channels)</td>
<td>8</td>
</tr>
<tr>
<td>k (radios per device)</td>
<td>3</td>
</tr>
<tr>
<td>$\tau(1)$ (max. throughput)</td>
<td>54 Mbps</td>
</tr>
</tbody>
</table>

![Graph showing efficiency over time]